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Quantum Physics of Nanostructures - Problem Set 4

Winter term 2022/2023

Due date: The problem set will be discussed Wednesday, 11.01.2023, 15:15-16:45, Room 114.

8. Hall resistance of the $\nu = 1/3$ state in the composite fermion picture

1 + 3 + 3 Points

Consider a Hall bar subjected to an external magnetic field $\boldsymbol{B}_{ext} = B\hat{z}$, Fig (1). In the fractional quantum Hall regime with filling factor $\nu = 1/3$, there are three flux quanta $\Phi_0 = h/e$ for each electron. In the lectures we have seen that an analogy between the integer and the fractional quantum Hall effect can be drawn by introducing the concept of composite fermions, where two flux quanta are attached to each electron. For $\nu = 1/3$, this is equivalent to an integer quantum Hall effect of the composite fermions with effective filling $\nu_{eff} = 1$. The aim of this problem is to compute the hall resistance R_H within the composite fermion picture.

(a) Since the composite fermions carry a flux, a finite density of them correspond to a magnetic field $\boldsymbol{B}_{CF}$. Compute the magnitude of this field in terms of the density of composite fermions.

(b) A current of composite fermions corresponds to a magnetic field moving at velocity v, which by the law of induction causes an electromotive force (emf) $\mathcal{E} = -\dot{\Phi}$. Here, the minus sign indicates that the emf tries to act against the effect which causes it. The actual direction of the emf should be determined via the Biot-Savart law. Measuring the Hall voltage is equivalent to measuring the induced emf of a conducting loop moving with a velocity $-v$ relative to a Hall bar with composite fermions at rest. Compute the Hall voltage due to the composite fermion current.

(c) Use the results obtained from part (a) to compute the total Hall resistance $R_H = V_H/I$. Note that besides the contribution computed in (b), there is a contribution of the charged composite fermions moving in an effective magnetic field corresponding to a filling fraction $\nu_{eff} = 1$. In particular, pay attention to whether the two contributions to the Hall voltage add up or partially cancel each other.

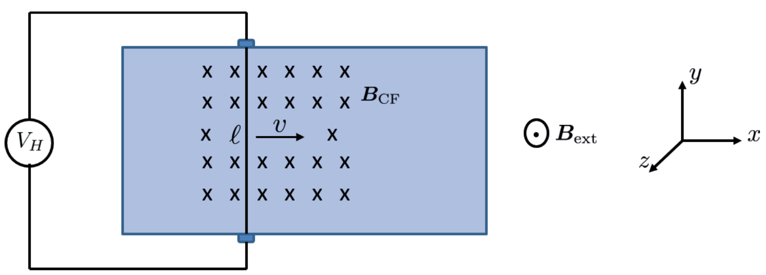

Abbildung 1: Schematic diagram of Hall bar setup

9. Anyons and the Aharonov-Bohm effect

3+3 Points

Consider a two-dimensional electron gas in the presence of a magnetic field. The conductivity tensor is given by

$$\sigma = \begin{pmatrix} 0 & \sigma_{xy} \\ -\sigma_{xy} & 0 \end{pmatrix},$$

where $\sigma_{xy} = \nu e^2/h$, with $0 < \nu < 1$, is the Hall conductivity.

(a) Suppose now a flux Φ is turned on adiabatically. Using Faraday's law and that the current density is given by $\boldsymbol{J} = \sigma \boldsymbol{E}$, where $\boldsymbol{E}$ is the induced electric field, show that the charge satisfies $\dot{Q} = \sigma_{xy}\dot{\Phi}$. How does the charge change if the flux changes by $\Phi_0 = h/e$?

(b) Now consider the composite object (quasiparticle) of a flux Φ_0 and charge $q = \nu e$. Determine the mutual statistics of these quasi particles. When do these composite objects behave as electrons? What do you get for $\nu = 1/3$ and $\nu = 1/5$? These states have been observed in experiments.

Hint: The exchange of the two quasi particles corresponds to moving one quasi particle by half a circle and performing a translation. This suggests that the wave function acquires a phase, which is half of the Berry phase acquired by a charge $q = \nu e$ moving along a path enclosing a magnetic flux Φ_0. Use this reasoning to obtain the exchange statistics of the composite objects for different values of ν. Quasi particles which acquire a phase different from 0 (bosons) or π (fermions) in the exchange are called anyons.

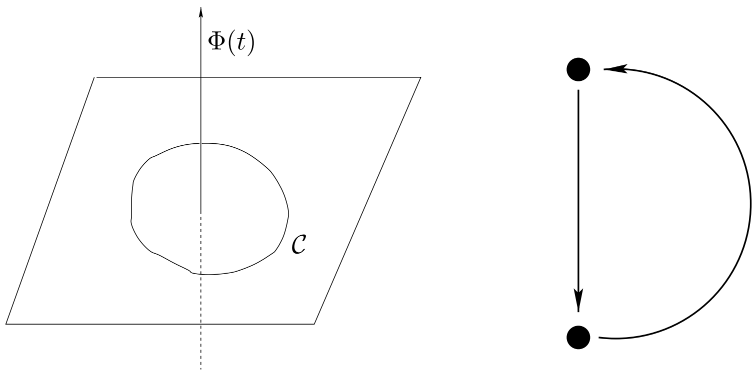

Abbildung 2: Left: The composite object is made up of a flux enclosed by a path $\mathcal{C}$ and a charge. Right: Illustration of how to exchange two quasi particles.